In-class Exercises to the Lecture Logics
Sheet 7

Jun.-Prof. Dr. Roland Meyer                    Discussion on July 18 and 19, 2013

**Exercise 7.1** [Resolution]

Using resolution, show that the formula

$$\forall z_1[q(z_1)] \vee \neg\forall x[(q(x) \vee r(x)) \wedge \exists z_2[\neg p(z_2) \wedge (p(z_2) \vee \neg r(x))]]$$

is a tautology. This amounts to

a) negating the formula,

b) bringing the result into clause form (Skolem + KNF), and

c) applying resolution to the formula in clause form.

**Exercise 7.2** [Calculating MGU]

For each of the following sets of literals, decide whether it is unifiable and if so, determine a most general unifier.

a) $\{q(x, z), q(h(y, z), f(a)), q(h(f(b), z), z)\}$.

b) $\{p(x, f(y)), p(f(a), y)\}$.

**Exercise 7.3** [An application to graphs]

By a *graph*, we mean an undirected (not necessarily finite) graph that may have loops.

a) Formalize the following statement as a formula in first order predicate logic: *If every node has a loop or has at least one other node that it is connected to, then every node is connected with an edge.*

b) Negate the formula and transform the result into an equisatisfiable formula without "=" (see In-Class Exercise 5.3).

c) Using resolution, show that the obtained formula is unsatisfiable.