Exercises to the lecture
Semantics
Sheet 3

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Exercise 3.1 (A parity game equivalent to HOMC)
Remember the APTA $\mathcal{A} = (\Sigma, Q, \delta, q_0, \Omega)$ from last sheet with $\Sigma = \{a : \sigma \to \sigma \to \sigma, b : \sigma\}$, $Q = \{q_0, q_1\}$, $\Omega = \{(q_0, 2), (q_1, 1)\}$ and transition relation

$$\begin{aligned} \delta(q_0, a) &= (1, q_1) \wedge (1, q_0) \wedge (2, q_0) \\ \delta(q_1, a) &= (1, q_1) \\ \delta(q_0, b) &= tt \end{aligned}$$

and the type judgement

$$\{H : (\theta_{H,q_0}, 2)^f, H : (\theta_{H,q_1}, 2)^f, F : (\theta_F, 2)^f\} \vdash \lambda x \lambda y.a(xyy)(F(Hx)y) : \theta_F.$$

where

$$\begin{aligned} \theta_{x,q_0} &= \bigwedge\{(q_0, 2), (q_1, 2)\} \to \bigwedge\{(q_0, 2), (q_1, 2)\} \to q_0, \\ \theta_{x,q_1} &= \bigwedge\{(q_0, 2), (q_1, 1)\} \to \bigwedge\{(q_0, 2), (q_1, 2)\} \to q_1, \\ \theta_{H,q_0} &= \bigwedge \{(\theta_{x,q_0}, 2), (\theta_{x,q_1}, 2)\} \to \theta_{x,q_0}, \\ \theta_{H,q_1} &= \bigwedge \{(\theta_{x,q_0}, 2), (\theta_{x,q_1}, 2)\} \to \theta_{x,q_1}, \\ \theta_F &= \bigwedge \{(\theta_{x,q_0}, 2), (\theta_{x,q_1}, 2)\} \to \bigwedge\{(q_0, 2), (q_1, 2)\} \to q_0. \end{aligned}$$

Consider the HORS $\mathcal{G} = (\Sigma, \mathcal{N}, \mathcal{R}, S)$ with

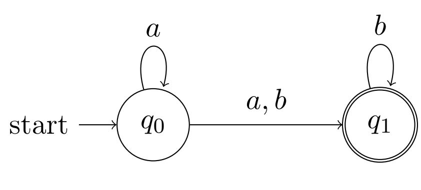
$$\mathcal{N} = \{S : \sigma, F : (\sigma \to \sigma \to \sigma) \to \sigma \to \sigma, H : (\sigma \to \sigma \to \sigma) \to \sigma \to \sigma \to \sigma\}$$

and

$$\begin{aligned} \mathcal{R}(S) &\coloneqq Fac \\ \mathcal{R}(H) &\coloneqq \lambda x.\lambda y.\lambda z.x(xyz)(xyz) \\ \mathcal{R}(F) &\coloneqq \lambda x.\lambda y.a(xyy)(F(Hx)y). \end{aligned}$$

What is the tree generated by $\mathcal{G}$?
Show that Eve has a winning strategy for the game $\mathbb{G}_{\mathcal{A},\mathcal{G}}$ from position $(S, q_0, 2)$.

Exercise 3.2 (NBA Complementation)
Consider the NBA $\mathcal{A}$ over $\Sigma = a, b$ below:



Use Büchi's complementation method discussed in class to compute $L(\mathcal{A})$ and $\overline{L(\mathcal{A})}$.