	Theoretical Computer Science 1	
Thomas Haas	**Exercise 1**	TU Braunschweig
Prof. Dr. Roland Meyer		Winter semester2020/21

Release date: 03.11.2020 Due date: 13.11.2020, 17:00

Hand in your solutions per E-Mail to your tutor until Friday, 13.11.2020 17:00 pm. You should provide your solutions either directly as .pdf file or as a readable scan/photo of your handwritten notes.

Aufgabe 1: Lattices [10 points]

a) [4 points] Let $(D_1, \preceq_1)$ and $(D_2, \preceq_2)$ be complete lattices. The **product lattice** is defined as $(D_1 \times D_2, \preceq)$, where $\preceq$ is the **product ordering** on tuples with $(d_1, d_2) \preceq (d'_1, d'_2)$ if and only if $d_1 \preceq_1 d'_1$ and $d_2 \preceq_2 d'_2$.

   Show that the product lattice is indeed a complete lattice.

b) [4 points] Prove the following; The product lattice $(D_1 \times D_2, \preceq)$ satisfies ACC (ascending chain condition) if and only if $(D_1, \preceq_1)$ and $(D_2, \preceq_2)$ both satisfy ACC.

c) [2 points] Give Hasse-Diagrams for lattices which:

   - are infinite but have bounded height.
   - have finite but non-bounded height.

Aufgabe 2: Distributivity [4 points]

Let $(D, \leqslant)$ be a lattice and $x, y \in D$ be two arbitrary elements..

a) [2 points] Show that if $f : D \to D$ is monotone, then $f(x \sqcup y) \geqslant f(x) \sqcup f(y)$ holds.

b) [2 points] $f : D \to D$ is called **distributive**, if $f(x \sqcup y) = f(x) \sqcup f(y)$ for all $x, y \in D$.
   Show that if f is distributive then f is also monotone.

Aufgabe 3: Reaching-Definitions-Analysis [10 points]

Note: The Reaching-Definitions-Analysis is content of next week's lecture.

Perform a Reaching-Definitions-Analysis on the following program.

$[x := 5]^1$
while $[x < 7]^2$ **do**
  $[y := y - 1]^3$
  if $[y = 7]^4$ **then**
    $[y := y + 3]^5$
  else
    $[x := x - 1]^6$
  end if
end while
$[skip]^7$

a) [2 points] Draw the control flow graph G.

b) [3 points] Consider the lattice $\mathcal{D} = (\mathcal{P}(\{x, y\} \times (\{1, \ldots, 6\} \cup \{?\})), \subseteq)$. For each of the blocks 1 – 6 give a suitable, monotone transfer function over this lattice.

c) [5 points] Consider the data flow system $(G, (D, \leqslant), \{(x, ?), (y, ?)\}, \text{TF})$, where TF is the set of transfer functions from part b). Write down the induced equation system and determine its least solution using Kleene's fixed-point theorem.

Aufgabe 4: Data flow analyses[10 points]

Note: These data flow analyses are NOT content of next week's lecture. However, next week's lecture will show how to apply Kleene's theorem to solve equation systems.

Consider the following program.

$[x := 3]^1$
$[x := x + 7]^2$
while $[x < 25]^3$ **do**
  $[x := x + 4]^4$
end while
$[skip]^5$

a) [2 points] Draw the control flow graph G.

b) [4 points] Consider the lattice $\mathcal{D} = (\mathbb{N} \cup \{\bot, \top\}, \preceq)$ with $\bot \preceq n \preceq \top$ ($\forall n \in \mathbb{N}$) from task 4a) of the first exercise sheet.

We interpret the lattice elements as data flow values with the following meanings:

$\bot$: Variable x is not initialized at the beginning of the block.

$n \in \mathbb{N}$: Variable *x* has constant value *n* at the beginning of the block (hence it is guaranteed to have value *n*).

$\top$: Variable *x* is not constant at the beginning of the block.

Using this lattice, perform a forward data flow analysis :

- Establish the data flow system $(G, \mathcal{D}, \bot, \mathrm{TF})$. For this, give monotone transfer functions for the blocks 1 – 5.
- Give the induced equation system.
- Find its least solution.

c) [4 points] Now consider the same program but a different lattice $\mathcal{D}' = (\mathcal{P}(\{e, o\}), \subseteq)$.

We interpret the lattice elements as data flow values with the following meanings:

$\emptyset$: Variable *x* is not initialized at the beginning of the block.

$\{e\}$: Variable *x* is guaranteed to be *even* at the beginning of the block.

$\{o\}$: Variable *x* is guaranteed to be *odd* at the beginning of the block.

$\{e, o\}$: It is not clear if variable *x* is even or odd at the beginning of the block.

Using this lattice, perform a forward data flow analysis:

- Establish the data flow system $(G, \mathcal{D}', \emptyset, \mathrm{TF})$. For this, give monotone transfer functions for the blocks 1 – 5.
- Give the induced equation system.
- Find its least solution.

Note: Unlike most analyses from the lecture, these two analysis are not expressible via the kill-gen-framework.